

## Chem 249 Problem Set 2

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### Three Phase Fresnel Calculation

1. Now that you've learned how to do it for two phases, create an Igor Pro file to calculate the Reflectivity for p-polarized and s-polarized light ( $R_p$  and  $R_s$ ) as a function of incident angle from normal incidence (0 degrees) to grazing incidence (90 degrees) in 0.1 degree increments for the following THREE phase system where the index of refraction  $n_1$  is real, and the indices of refraction of  $n_2$  and  $n_3$  can be complex:

i)  $n_1 := 1.51$  (silica)

ii)  $n_2 := 0.18 + 3.40i$  (gold)

iii)  $n_3 := 1.00 + 0i$  (air)

iv)  $\lambda = 633$  nm

v)  $h :=$  initial value 45.0, vary  $h$  in a box by increments of 1 nm.

See the Hansen Paper for detailed equations. For Igor Pro, a text file of the refined History emailed to me is the most useful, along with an initial print out.

### Frequency Domain Fluorescence Measurements

2. Frequency domain fluorescence spectroscopy (fdfs) is an alternative method for obtaining fluorescence lifetimes. In these measurement, two parameters: a phase shift angle ( $\phi$ ) and demodulation factor ( $m$ ), are measured. What is the mathematical relationship between these measured parameters and the fluorescence lifetime ( $\tau$ )? How is  $\tau$  extracted from the fdfs data?

### Classical Simple Harmonic Oscillator and LaPlace Transforms

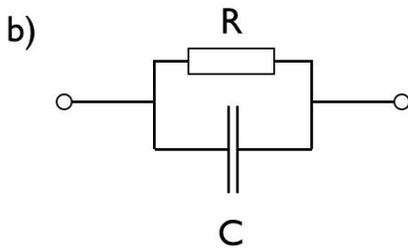
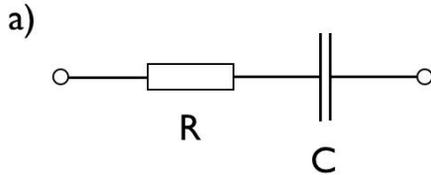
3. Consider the equation of motion for a damped harmonic oscillator:

$$m \frac{d^2 x}{dt^2} + 2m\Gamma \frac{dx}{dt} + m\omega^2 x = 0$$

with the initial conditions  $x(0) = X_0$  and  $x'(0) = V_0$ . Use Laplace transforms to find  $x(t)$  for the two cases where  $\omega \gg \Gamma$  and  $\omega = \Gamma$ . Make plots in Igor of a couple of sample trajectories for these two cases.

## Frequency Analysis: Nyquist and Bode Plots

4. Generate a Nyquist Impedance Plot and Bode Impedance plots (both magnitude and phase) for an angular frequency range of  $10^{-2}$  rad s<sup>-1</sup> to  $10^8$  rad s<sup>-1</sup> for these two RC circuits:



Where  $R = 1000$  ohms and  $C = 2.0$   $\mu$ F.

## Complex Frequency Dependent Susceptibility: Optical Absorption Spectrum

5. Consider the classical equation of motion for an electron in a damped harmonic well:

$$m \frac{d^2 x}{dt^2} + 2m\Gamma \frac{dx}{dt} + m\omega_0^2 x = F(t)$$

where  $F(t)$  is an externally applied Force. If  $F(t) = qE(t) = -eE_0 \sin(\omega t)$ , then the frequency dependent Polarization  $P(\omega) = Ne x(\omega) = \chi(\omega) E(\omega)$  where  $N$  is the number of oscillators/volume, and  $\chi(\omega)$  is called the frequency dependent electric susceptibility and has the form:

$$\chi(\omega) = \frac{S\omega_0^2}{\omega_0^2 - \omega^2 + i\omega\Gamma}$$

a) Please generate a plot of  $\text{Im}\chi$  versus  $\omega$  (a kind of Bode plot) for three cases where  $S = 0.1$ ,  $\omega_0 = 1000$ , and  $\Gamma = 10, 50, 100$ . Measure the FWHM (full width at half max) for the resonant peak. How does it scale with  $\Gamma$ ?

b) Please generate a Nyquist plot ( $\text{Im}\chi$  vs  $\text{Re}\chi$ ) for the case  $S = 0.1$ ,  $\omega_0 = 1000$ , and  $\Gamma = 50$ .

6. Using the general form for the general form for the electric susceptibility where  $S = 0.1$ ,  $\omega_0 = 1000$ , and  $\Gamma = 20$ , please plot the real and imaginary parts of  $n$ , the complex index of refraction ( $n = n_r + in_i$ ), versus omega. Recall that  $n^2 = \epsilon_r = 1 + \chi$ .