

Two coupled Spin 1/2 particles.  
 R. Corn, March 2014.

$$S_{tot}^2 = S_1^2 + S_2^2 + 2S_1 \cdot S_2$$

$$S_{tot}^2 = S_1^2 + S_2^2 + 2S_{1z}S_{2z} + (S_1^+S_2^- + S_1^-S_2^+)$$

$$S_{tot}^2 |++\rangle = \left(\frac{3}{4} + \frac{3}{4} + \frac{1}{2}\right) \hbar^2 |++\rangle = 2\hbar^2 |++\rangle$$

$$S_{tot}^2 |--\rangle = \left(\frac{3}{4} + \frac{3}{4} + \frac{1}{2}\right) \hbar^2 |--\rangle = 2\hbar^2 |--\rangle$$

$$S_{tot}^2 |+-\rangle = \left(\frac{3}{4} + \frac{3}{4} - \frac{1}{2}\right) \hbar^2 |+-\rangle + \hbar^2 |-+\rangle = \hbar^2 |+-\rangle + \hbar^2 |-+\rangle$$

$$S_{tot}^2 |-+\rangle = \left(\frac{3}{4} + \frac{3}{4} - \frac{1}{2}\right) \hbar^2 |-+\rangle + \hbar^2 |+-\rangle = \hbar^2 |-+\rangle + \hbar^2 |+-\rangle$$



Eigenvalues{{2,0,0,0},{0,1,1,0},{0,1,1,0},{0,0,0,2}}

Input:

$$\text{Eigenvalues}\left[\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}\right]$$

Results:

$$\lambda_1 = 2$$

$$\lambda_2 = 2$$

$$\lambda_3 = 2$$

$$\lambda_4 = 0$$

Corresponding eigenvectors:

$$v_1 = (0, 0, 0, 1)$$

$$v_2 = (0, 1, 1, 0)$$

$$v_3 = (1, 0, 0, 0)$$

$$v_4 = (0, -1, 1, 0)$$

New (j,m) Eigenstates:

$$|1,1\rangle = |++\rangle$$

$$|1,0\rangle = \frac{1}{\sqrt{2}}(|+-\rangle + |-+\rangle)$$

$$|1,-1\rangle = |--\rangle$$

$$|0,0\rangle = \frac{1}{\sqrt{2}}(|+-\rangle - |-+\rangle)$$

The j=1 states are the "triplet" states, and the j=0 state is the "singlet" state.

Both the (+,+) basis and the (j,m) basis are eigenstates of  $H_0$ .

But only the (j, m) basis are eigenstates for W and thus H (=H<sub>0</sub> +W).

These equations are for two identical spins ( $\omega_a = \omega_b = \omega_0$ ).

If  $\omega_a \neq \omega_b$ , then neither basis are eigenstates.