Two coupled Spin 1/2 particles.
R. Corn, March 2014.

$$S_{tot}^2 = S_1^2 + S_2^2 + 2S_1 \bullet S_2$$

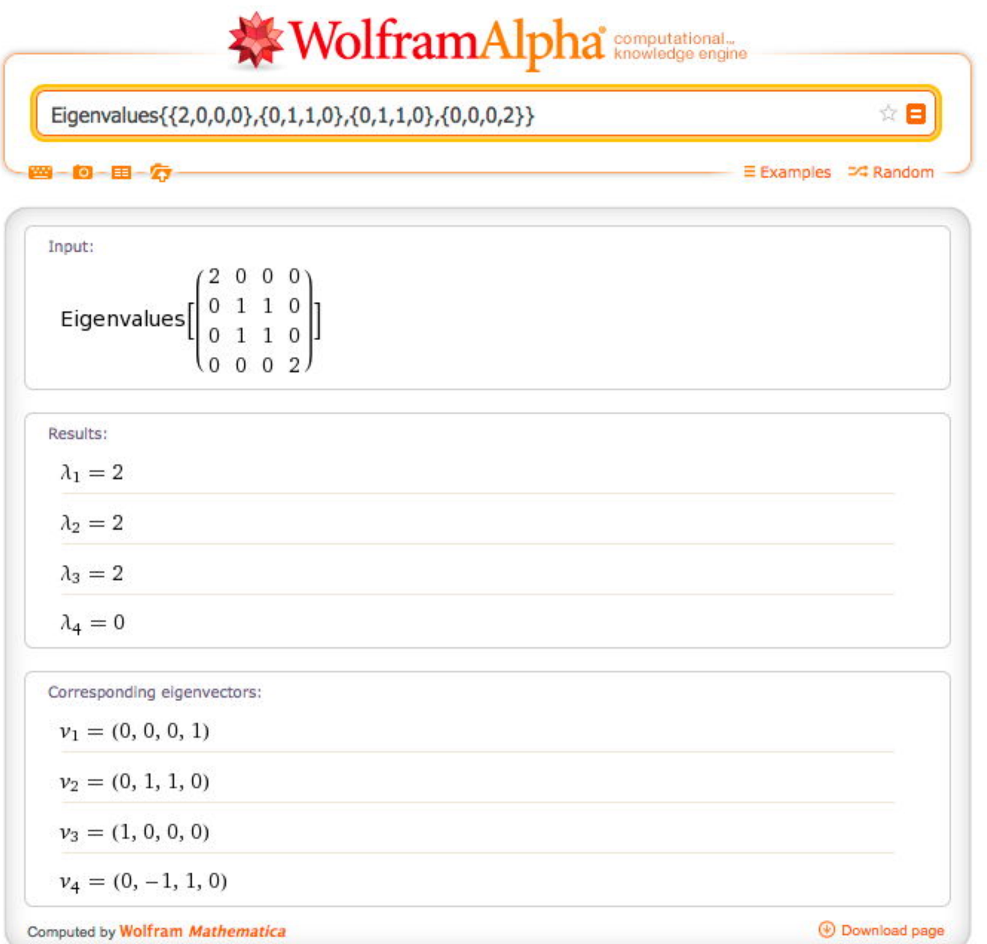
$$S_{tot}^2 = S_1^2 + S_2^2 + 2S_{1z}S_{2z} + \left(S_1^+S_2^- + S_1^-S_2^+\right)$$

$$S_{tot}^2\left|++\right\rangle = \left(\frac{3}{4}+\frac{3}{4}+\frac{1}{2}\right)\hbar^2\left|++\right\rangle = 2\hbar^2\left|++\right\rangle$$

$$S_{tot}^2\left|--\right\rangle = \left(\frac{3}{4}+\frac{3}{4}+\frac{1}{2}\right)\hbar^2\left|--\right\rangle = 2\hbar^2\left|--\right\rangle$$

$$S_{tot}^2\left|+-\right\rangle = \left(\frac{3}{4}+\frac{3}{4}-\frac{1}{2}\right)\hbar^2\left|+-\right\rangle + \hbar^2\left|-+\right\rangle = \hbar^2\left|+-\right\rangle + \hbar^2\left|-+\right\rangle$$

$$S_{tot}^2\left|-+\right\rangle = \left(\frac{3}{4}+\frac{3}{4}-\frac{1}{2}\right)\hbar^2\left|-+\right\rangle + \hbar^2\left|+-\right\rangle = \hbar^2\left|-+\right\rangle + \hbar^2\left|+-\right\rangle$$



New (j,m) Eigenstates:

$$|1,1\rangle = |++\rangle$$
$$|1,0\rangle = \frac{1}{\sqrt{2}}\left(|+-\rangle + |-+\rangle\right)$$
$$|1,-1\rangle = |--\rangle$$
$$|0,0\rangle = \frac{1}{\sqrt{2}}\left(|+-\rangle - |-+\rangle\right)$$

The j=1 states are the "triplet" states, and the j=0 state is the "singlet" state.

Both the (+,+) basis and the (j,m) basis are eigenstates of H_o.
But only the (j, m) basis are eigenstates for W and thus H (=H_o +W).

These equations are for two identical spins ($\omega_a = \omega_b = \omega_0$).

If $\omega_a \neq \omega_b$, then neither basis are eigenstates.