Derivation of the Equation of Motion for a Spin System (Rabi Paper).
R. Corn, March 2014.

$$H = -\mu \bullet \vec{B} = -\gamma \vec{I} \bullet \vec{B}$$

$$\frac{d\vec{I}}{dt} = \frac{i}{\hbar}\left[H, \vec{I}\right] = \frac{i}{\hbar}\left(H\vec{I} - \vec{I}H\right)$$

$$\frac{d\vec{I}}{dt} = -\frac{i\gamma}{\hbar}\left(\left(\vec{I} \bullet \vec{B}\right)\vec{I} - \vec{I}\left(\vec{I} \bullet \vec{B}\right)\right)$$

Look at the x-component of **I**:

$$\frac{dI_x}{dt} = -\frac{i\gamma}{\hbar}\left(\left(I_xB_x + I_yB_y + I_zB_z\right)I_x - I_x\left(I_xB_x + I_yB_y + I_zB_z\right)\right)$$

$$\frac{dI_x}{dt} = -\frac{i\gamma}{\hbar}\left(\left(I_yI_xB_y + I_zI_xB_z\right) - \left(I_xI_yB_y + I_xI_zB_z\right)\right)$$

$$\frac{dI_x}{dt} = -\frac{i\gamma}{\hbar}\left(\left(I_yI_x - I_xI_y\right)B_y - \left(I_zI_x - I_xI_z\right)B_z\right)$$

$$\frac{dI_x}{dt} = \gamma\left(I_zB_y - I_yB_z\right)$$

where we have used:

$$\left[I_x, I_y\right] = i\hbar I_z$$

Similar equations for the y and z components give us three equations that can be expressed in total as:

$$\frac{d\vec{I}}{dt} = \gamma\vec{I} \times \vec{B}$$

This is equation (1) in the Rabi paper.