Landé g-factor Derivation.
R. Corn -- March 2014.

In the weak field limit, we assume that the magnetic dipole moment of the atom is proportional to the total angular momentum **J**:

$$\vec{\mu}_J = g_J m_B \vec{J}$$

where m_B is the Bohr magneton and g_J is the Landé g factor for **J**. To find a value of g_J, we relate **J** to the known values of g_L and g_S and assume the following:

$$\vec{J} = \vec{L} + \vec{S}$$
$$\vec{\mu}_J = \vec{\mu}_L + \vec{\mu}_S$$
$$\vec{\mu}_L = g_L m_B \vec{L}$$
$$\vec{\mu}_S = g_S m_B \vec{S}$$
$$\vec{\mu}_J = g_J m_B \vec{J} = g_L m_B \vec{L} + g_S m_B \vec{S}$$
$$g_J \vec{J} = g_L \vec{L} + g_S \vec{S}$$

We then take the dot product with **J** and get:

$$g_J \vec{J} \bullet \vec{J} = g_L \vec{L} \bullet \vec{J} + g_S \vec{S} \bullet \vec{J}$$
$$g_J J^2 = g_L \left(L^2 + \vec{L} \bullet \vec{S}\right) \vec{L} \bullet \vec{J} + g_S \left(S^2 + \vec{L} \bullet \vec{S}\right)$$
$$g_J J^2 = g_L \left(L^2 + 0.5\left(J^2 - L^2 - S^2\right)\right) + g_S \left(S^2 + 0.5\left(J^2 - L^2 - S^2\right)\right)$$
$$g_J J(J+1) = 0.5 g_L \left(J(J+1) + L(L+1) - S(S+1)\right) + 0.5 g_S \left(\left(J(J+1) - L(L+1) + S(S+1)\right)\right)$$
$$g_J = \frac{g_L \left(J(J+1) + L(L+1) - S(S+1)\right) + g_S \left(\left(J(J+1) - L(L+1) + S(S+1)\right)\right)}{2J(J+1)}$$

For the case:

$$g_L = 1$$
$$g_S = 2$$

We get:

$$g_J = \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)}$$

And so, for the weak field limit:

$$\vec{\mu}_J = g_J m_B \vec{J}$$
$$W_Z = g_J \omega_0 J_z$$

$g_J = 4/3$ for 2p(j=3/2)

$g_J = 2/3$ for 2p(j=1/2)

where ω_0 is the Larmor angular frequency:

$\omega_0 = -eB_0/2m_e$.

Et voilà! These are eqns. 11-13 in the CTcompD12.pdf handout.