

Two Phase Fresnel Equations from the Hansen Paper.  
(W. N. Hansen, JOSA 58, 380-390 (1968))  
Chem 249, R. Corn.

**Variables:**

|                            |   |
|----------------------------|---|
| $n_1(\omega)$              | real refractive index for phase 1                               |
| $n_2(\omega)$              | complex refractive index for phase 2                            |
| $\epsilon_1(\omega)$       | real dielectric constant for phase 1                            |
| $\epsilon_2(\omega)$       | complex dielectric constant for phase 2                         |
| $r_s(\omega, \theta)$      | complex Fresnel coefficient for s-polarized light               |
| $r_p(\omega, \theta)$      | complex Fresnel coefficient for p-polarized light               |
| $\delta_s(\omega, \theta)$ | phase shift for the reflected s-polarized light (=arg( $r_s$ )) |
| $\delta_p(\omega, \theta)$ | phase shift for the reflected p-polarized light (=arg( $r_p$ )) |
| $R_s(\omega, \theta)$      | Reflection coefficient for s-polarized light                    |
| $R_p(\omega, \theta)$      | Reflection coefficient for p-polarized light                    |

**Two phase Fresnel Calculation:** (incident angle is denoted as  $\theta$  or  $\theta_1$ )

$$\xi_1(\omega, \theta) = n_1 \cos \theta_1$$

$$\xi_2(\omega, \theta) = n_2 \cos \theta_2 = (n_2^2 - n_1^2 \sin^2 \theta_1)^{1/2} \quad \xi_2 \text{ can be complex!}$$

$$r_s = |r_s| e^{i\delta_s} = \frac{\xi_1 - \xi_2}{\xi_1 + \xi_2}$$

$$r_p = |r_p| e^{i\delta_p} = \frac{\epsilon_2 \xi_1 - \epsilon_1 \xi_2}{\epsilon_2 \xi_1 + \epsilon_1 \xi_2}$$

$$R_s = |r_s|^2$$

$$R_p = |r_p|^2$$

Try to make plots of  $R_s$ ,  $R_p$ ,  $\delta_s$  and  $\delta_p$  versus incident angle  $\theta = 0^\circ$  to  $90^\circ$  where you can input  $n_1$  and  $n_2$  (remember that  $n_2$  is complex).

## Hansen Two Phase Calculation: Interfacial Electric Fields.

The average electric fields at an interface have components in the x, y and z directions, Where z is normal to the interface, x is in the plane of incidence, and y is perpendicular to the plane of incidence. S-polarized light has only an  $E_y$  component; p-polarized light has an  $E_z$  and an  $E_x$  component.

Fields can be calculated either just above the interface (in phase 1) or just below the interface (in phase 2).

Just above the interface:

$$\begin{aligned}\langle E_y^2 \rangle &= \left[ \frac{1}{2}(1 + R_s) + R_s^{\frac{1}{2}} \cos \delta_s \right] \langle E_{in}^2 \rangle \\ \langle E_z^2 \rangle &= \sin^2 \theta \left[ \frac{1}{2}(1 + R_p) + R_p^{\frac{1}{2}} \cos \delta_p \right] \langle E_{in}^2 \rangle \\ \langle E_x^2 \rangle &= \cos^2 \theta \left[ \frac{1}{2}(1 + R_p) - R_p^{\frac{1}{2}} \cos \delta_p \right] \langle E_{in}^2 \rangle\end{aligned}$$

where the complex Fresnel reflection coefficients  $r_s$  and  $r_p$  are:

$$\begin{aligned}r_s &= R_s^{\frac{1}{2}} \exp(i\delta_s) \\ r_p &= R_p^{\frac{1}{2}} \exp(i\delta_p)\end{aligned}$$

If we let  $E_{in} = 1$ , then  $\langle E_{in}^2 \rangle = \frac{1}{2}$ .

Just below the interface:

$$\begin{aligned}\langle E_y^2 \rangle &= \frac{1}{2} |t_s|^2 \langle E_{in}^2 \rangle \\ \langle E_z^2 \rangle &= \frac{1}{2} \left| \frac{n_1 \sin \theta}{n_2} t_p \right|^2 \langle E_{in}^2 \rangle \\ \langle E_x^2 \rangle &= \frac{1}{2} \left| \frac{\xi_2}{n_2} t_p \right|^2 \langle E_{in}^2 \rangle\end{aligned}$$

where the complex Fresnel transmission coefficients  $t_s$  and  $t_p$  are:

$$t_s = \frac{2\xi_1}{\xi_1 + \xi_2}$$

$$t_p = \frac{n_1}{n_2} \left[ \frac{2\varepsilon_2 \xi_1}{\varepsilon_2 \xi_1 + \varepsilon_1 \xi_2} \right]$$

Remember that  $n_2$  can be complex!