The Electric Susceptibility, Dielectric Constant, and Complex Index of Refraction
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Electric Polarization: $\mathbf{P}(\omega) = \varepsilon_0 \chi(\omega) \mathbf{E}(\omega)$

Electric Displacement: $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$

$$\mathbf{D} = \varepsilon_0 (1 + \chi) \mathbf{E} = \varepsilon_0 \varepsilon_r(\omega) \mathbf{E} = \varepsilon \mathbf{E}$$

$\chi(\omega)$ = complex frequency dependent electric susceptibility

ε_0 = permittivity of free space
ε = permittivity
$\varepsilon_r(\omega)$ = relative permittivity or complex frequency dependent dielectric constant

$\chi = \chi' + j\chi''$
$\varepsilon_r = 1 + \chi = (1 + \chi') + j\chi''$

EM Plane Wave: $\mathbf{E}(\mathbf{r}, t) = \mathbf{E_0} \exp(j\mathbf{k} \cdot \mathbf{r} - j\omega t)$

In free space: $k = \omega(\varepsilon_0 \mu_0)^{1/2} = \dfrac{\omega}{c}$

$$c = (\varepsilon_0 \mu_0)^{-1/2}$$

c = speed of light
μ_0 = permeability of free space

In a dielectric: $k = \omega(\varepsilon \mu_0)^{1/2} = \omega(\varepsilon_r \varepsilon_0 \mu_0)^{1/2} = \dfrac{n\omega}{c}$

$$n = \varepsilon_r^{1/2}$$

$n = \eta + j\kappa$
$\varepsilon_r = n^2 = (\eta^2 - \kappa^2) + j(2\eta\kappa)$
n = complex index of refraction
η = (real) refractive index
κ = extinction coefficient

EM wave in z direction: $E(z,t) = E_0 \exp\left(j\omega\left(\dfrac{\eta z}{c} - t \right) - \dfrac{\omega \kappa z}{c} \right)$

Beer's Law:

$$I(z) = I_0 \exp(-Kz)$$

$$K = \frac{2\omega\kappa}{c}$$

K = Beer's Law absorption coefficient