The resonant frequency is often expressed in natural units (radians per second), rather than using the f_0 in hertz, as

$$\omega_0 = 2\pi f_0 .$$

The factors Q, damping ratio ζ, attenuation rate α, and exponential time constant τ are related such that:[12]

$$Q = \frac{1}{2\zeta} = \frac{\omega_0}{2\alpha} = \frac{\tau\omega_0}{2},$$

and the damping ratio can be expressed as:

$$\zeta = \frac{1}{2Q} = \frac{\alpha}{\omega_0} = \frac{1}{\tau\omega_0}.$$

The envelope of oscillation decays proportional to $e^{-\alpha t}$ or $e^{-t/\tau}$, where α and τ can be expressed as:

$$\alpha = \frac{\omega_0}{2Q} = \zeta\omega_0 = \frac{1}{\tau}$$

and

$$\tau = \frac{2Q}{\omega_0} = \frac{1}{\zeta\omega_0} = \frac{1}{\alpha}.$$

The energy of oscillation, or the power dissipation, decays twice as fast, that is, as the square of the amplitude, as $e^{-2\alpha t}$ or $e^{-2t/\tau}$.

For a two-pole lowpass filter, the transfer function of the filter is[12]

$$H(s) = \frac{\omega_0^2}{s^2 + \underbrace{\frac{\omega_0}{Q}}_{2\zeta\omega_0 = 2\alpha} s + \omega_0^2}$$

For this system, when $Q > \frac{1}{2}$ (i.e., when the system is underdamped), it has two complex conjugate poles that each have a real part of $-\alpha$. That is, the attenuation parameter α represents the rate of exponential decay of the oscillations (that is, of the output after an impulse) into the system. A higher quality factor implies a lower attenuation rate, and so high-Q systems oscillate for many cycles. For example, high-quality bells have an approximately pure sinusoidal tone for a long time after being struck by a hammer.